PLUS ONE

Mathematics

Study material

(SCERT Focus Area Based)

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Jiju Murali

CHAPTER 1

SETS

A **set** is a well defined collection of objects. 'Well defined' means we can definitely decide whether a given particular object belongs to a given collection or not.

The objects in a set are called **elements**. The number of elements in a set is called its **cardinality** or **cardinal number**. We shall denote sets by capital alphabets A,B,C ,…

Methods to represent a set

1. Roster form (Tabular form) 2. Set builder form

1. **ROSTER FORM**

   In this form, we are listing the elements, separated by commas and enclose in braces {}. The elements can be written in any order.
   ie, {1,2,3} and {2,3,1} are same
   An element is not generally repeated

2. **SET BUILDER FORM**

   In this form, the characterizing property of the elements is stated.
   For this, a variable x (denotes each element of the set) is written inside the braces,and then the symbol ' : ' or ' | 'and following it the common property of the elements is stated.
   Eg:

   Consider the set of vowels in English alphabets

   The roster form is {a,e,i,o,u}
   The set builder form is {x : x is a vowel in English alphabet}

N : The set of all natural numbers
W : The set of all whole numbers
Z : The set of all integers
Z^+ : The set of all positive integers
Q : The set of all rational numbers
Q^+ : The set of all positive rational numbers

R : The set of all real numbers
R^+ : The set of all positive real numbers

∈ : Element of (belongs to)

∉ : Not an element of (not belongs to)

The symbols ∈,∉ used to denote whether an object is an element of a set.

If **x** is an element of the set **A**, then we write **x**∈**A**

If **x** is not an element of the set **A**, then we write **x**∉**A**

TYPES OF SETS

EMPTY SET : A set which does not contain any element is called **empty set** or **null set** or **void set**.

It is denoted by the symbol ϕ or **{ }**.

SINGLETON SET : A set consisting of a single element is called a **singleton set**

FINITE SET : A set which is empty or consists of a definite number of elements is called **finite set**.

The number of elements of a finite set **A** is denoted by **n(A) [**read as n of A].

Null set is a finite set.

INFINITE SET : A set which is not finite is called **infinite set.**

EQUAL SETS : Two sets A and B are called **equal sets** if every element of A is an element of B and every element of B is an element of A(they have exactly the same elements). If A and B are equal sets , we write A = B.

SUBSETS AND SUPER SETS

$\subset$: subset of(contained in)

$\supset$: super set of

Consider two sets A and B. If every element of A is an element of B, then A is a subset of B and it is written as $A \subset B$ (read as A is a subset of B).

We also say that B is a super set of A and is written as $B \supset A$(read as B is a super set of A).

Note

1. If $A \subset B$ and $B \subset A$, then $A = B$.
2. Every set is a subset of itself. ($A \subset A$)
3. Null set is a subset of every set.($\phi \subset A$)

$N \subset W \subset Z \subset Q \subset R$

The number of subsets of a set having **n** elements = $\mathbf{2^n}$

PROPER SUBSETS : The subsets of a set other than the set itself are called **proper subsets**

Number of Proper subsets of a set having **n** elements = $\mathbf{2^n-1}$

POWER SET : The set of all possible subsets of a given set **A** is called **power set** of A and is denoted by **P(A).** The number of elements in P(A) = $\mathbf{2^n}$

Operations on Sets

Union of Sets: Let A and B be two sets. The union of A and B is the set of all elements of A and all elements of B, the common elements being taken only once and it is denoted by **$A \cup B$** (read as A union B).

Intersection of Sets : Let A and B be two sets. The intersection of A and B is the set of all elements which are common to both A and B and it is denoted by **$A \cap B$** (read as A intersection B).

Disjoint sets : If **$A \cap B = \emptyset$**, A and B are called **disjoint sets**.

Note:

If $A \subset B$, Then $A \cup B = B$ and $A \cap B = A$

Difference of sets : **A-B**(A minus B) is the set of elements which belong to A but not to B. **B-A** (B minus A) is the set of elements which belong to B but not to A.

Note: **If A and B are disjoint sets, then A-B =A and B-A = B**

Universal set : A universal set is a set containing all elements of sets in a given context. Universal set is denoted by **U**.

Complement of a set : The complement of a set A is the set of elements in U **not in A**. The complement of A is denoted by A' or A^c

$A' = U - A$

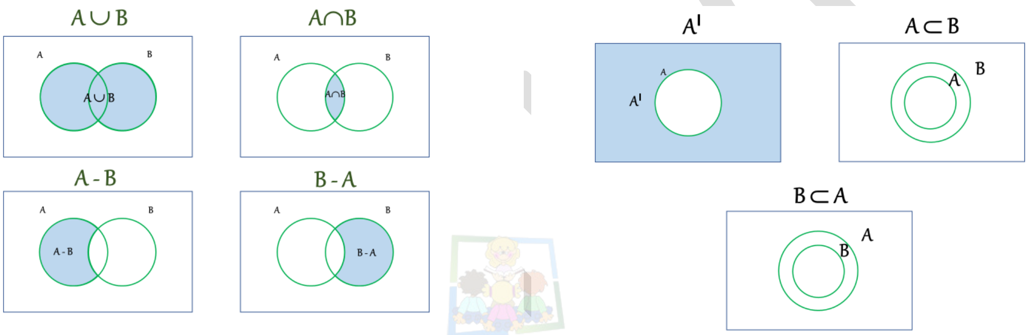
1. $(A')' = A$
2. $\phi' = U$
3. $U' = \phi$
4. $A \cup A' = U$
5. $A \cap A' = \phi$

De Morgan's Laws

1. **$(A \cup B)' = A' \cap B'$**
2. **$(A \cap B)' = A' \cup B'$**

VENN DIAGRAMS

A Venn Diagram is a diagrammatic representation of finite sets. Universal set is represented by rectangle and other sets by circles (or triangles).



Results on number of elements in sets

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. $n(A \cap B) = n(A) + n(B) - n(A \cup B)$
3. If A and B are disjoint sets, then $n(A \cup B) = n(A) + n(B)$
4. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Write the following set- builder forms into roster form

1. $\{x : x$ is a prime number less than $10\}$

   Ans: $\{2,3,5,7\}$
2. $\{x : x \in N, 2 \leq x < 5\}$

   Ans : $\{2,3,4\}$
3. $\{x : x \in Z, -2 \leq x \leq 3\}$

   Ans: $\{-2,-1,0,1,2,3\}$

4. { x : x is an integer, $\frac{1}{2} < x < \frac{9}{2}$ }
   Integers are ….-3, -2, -1, 0, 1, 2, 3, 4, 5 ….
   1/2 = 0.5 and 9/2 = 4.5
   -2 ,-1, 0 , **0.5,** 1,2 ,3, 4, **4.5**,5, 6
   Ans : {1,2,3,4}

5. {x : x∈R, x^2- 4 = 0 }

   $x^2 - 4 = 0$

   $x^2 = 4$
   $x = \sqrt{4}$
   = +2, -2
   Ans: {-2,2}

6. {x : x∈R, x^2- 5x + 6 = 0}
   x^2-5x+6 = 0
   It is a second degree equation
   a =1, b = -5, c = 6
   $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
   = 2,3
   Ans : {2,3}

7. { x : x is a positive divisor of 6}
   Ans : {1,2,3,6}

8. {x : x∈Z, |x|<3}
   {-2,-1,0,1,2}

2. A = { x : x is a prime number less than 7}

   a) Write A in roster form
      A = {2,3,5}
   b) Write P(A)
      P(A) = { {2},{3}, {5}, {2,3}, {3,5}, {2,5}, {2,3,5}, ϕ }

c) Write the number of proper subsets of A

2^n-1

$=2^3$-1

= 8 – 1

= 7

3. A = {x : x is an integer $-\frac{1}{2} < x < \frac{9}{2}$} , B = {x : x$\in$N, 2< x $\leq$ 5}

Verify that (A$\cup$B) - (A$\cap$B)= (A-B) $\cup$ (B-A)

Integers are …, -3, -2, -1, 0, 1, 2, 3, 4, 5,…

-3, -2, -1, -0.5, 0 , 1, 2, 3, 4, 4.5 ,5

A = {0,1,2,3,4}

B= {3,4,5}

L.H.S

A$\cup$B = {0,1,2,3,4} $\cup$ {3,4,5} = {0,1,2,3,4,5}

A$\cap$B = {0,1,2,3,4} $\cap$ {3,4,5} = {3,4}

(A$\cup$B) - (A$\cap$B) = {0,1,2,3,4,5} – {3,4} = **{0,1,2,5}**

R.H.S

A – B = {0,1,2,3,4} - {3,4,5} = {0,1,2}

B – A = {3,4,5} – {0,1,2,3,4} = {5}

(A-B) $\cup$ (B-A) = {0,1,2} $\cup$ {5} = **{0,1,2,5}**

L.H.S = R.H.S

4. U = {x : x$\in$W, x $\leq$ 6} , A = {x : x is a prime divisor of 6} , B = {1,2,5}

Verify De Morgan's laws

U = {0,1,2,3,4,5,6}

A = {2,3}

B = {1,2,5}

$A^{/}$ = U – A

= {0,1,2,3,4,5,6} – {2,3}

= {0,1,4,5,6}

$B' = U - B$

$= \{0,1,2,3,4,5,6\} - \{1,2,5\}$

$= \{0,3,4,6\}$

1. **$(A \cup B)' = A' \cap B'$**

   L H S

   $A \cup B = \{2,3\} \cup \{1,2,5\}$

   $= \{1,2,3,5\}$

   $(A \cup B)' = \{0,4,6\}$

   RHS

   $A' \cap B' = \{0,1,4,5,6\} \cap \{0,3,4,6\}$

   $= \{0,4,6\}$

   LHS = RHS

2. **$(A \cap B)' = A' \cup B'$**

   LHS

   $(A \cap B) = \{2,3\} \cap \{1,2,5\}$

   $= \{2\}$

   $(A \cap B)' = \{0,1,3,4,5,6\}$

   RHS

   $A' \cup B' = \{0,1,4,5,6\} \cup \{0,3,4,6\}$

   $= \{0,1,3,4,5,6\}$

   LHS = RHS

5. In a group of students 100 know Hindi, 50 know English and 33 know both. Each of the students know Hindi or English. How many students are there in the group?

Ans :

H : The set of students know Hindi

E : The set of students know English

$n(H) = 100$

$n(E) = 50$

$n(E \cap H) = 33$

$n(E \cup H) = n(E) + n(H) - n(A \cap H)$

$= 50 + 100 - 33$

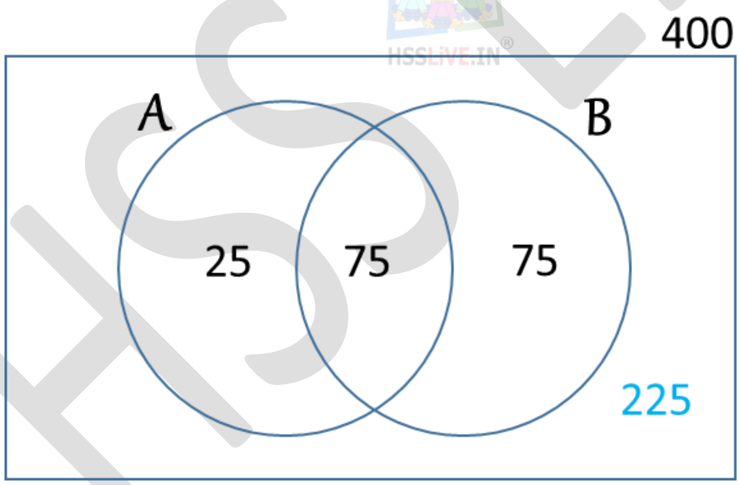
$= 117$

6. In a survey of 400 students, 100 like apple juice, 150 like orange juice and 75 like both. How many students like neither apple juice nor orange juice?

Ans:

A : The set of students like apple juice

B : The set of students like orange juice



$n(A) = 100$

$n(B) = 150$

$n(A \cap B) = 75$

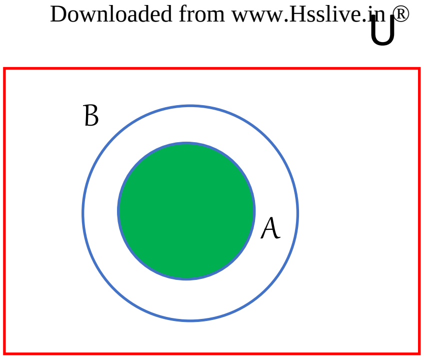
400-(25+75+75) = 400-175 = 225

Number of students like neither apple juice nor orange juice = **225**

7. If $A \subset B$, $A \cap B$ = ………. Also draw the Venn diagram of $A \cap B$

Ans:

$A \cap B = A$



8. n(A) = 5, n(B) = 4, find n(A-B) if A and B are disjoint

   Ans:

   If A and B are disjoint, A – B = A
   n(A-B) = n(A)
   = 5

9. The number of proper subsets of a set is 15. Find the number of elements of the set

   Ans :

   The number of proper subsets of a set = 15

   $2^n - 1 = 15$

   $2^n = 15 + 1 = 16$

   $2^n = 2^4$

   $n = 4$

Chapter 2

<u>Relations and Functions</u>

<u>Ordered pair</u>

A pair having a definite order is called ordered pair.

Consider (a,b). a is called first component and b is called second component.

If (a,b) = (c,d), the a = c and b = d

Eg : If (2x-3, 3y+1) = (5,7), find x and y

Ans:

Equating the first component

$2x-3 = 5$

$2x = 5 + 3 = 8$

$x = 8/2 = 4$

equating the second component

$3y+1 = 7$

$3y = 7-1 = 6$

$y = 6/3 = 2$

Cross product (Cartesian Product)

A × B is the set of all ordered pairs in which first element is from A and second element from B

Q. Let A = {1,2,3} and B = {4,5}, write A×B and B×A?

Ans:

A×B = {(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)}

B×A = {(4,1),(4,2),(4,3),(5,1),(5,2),(5,3)}

Note : If $A = \phi$ or $B = \phi$, then $A\times B = \phi$

$n(A\times B) = n(A).n(B)$

$n(A) = 3$, $n(B) = 2$

$n(A\times B) = 3\times 2 = 6$

$n(A\times A) = n(A).\ n(A) = 3\times 3 = 9$

$n(A\times A\times A) = n(A).\ n(A)\ .n(A) = 3\times 3\times 3 = 27$

Q. A = {-1,0}. Find A×A×A

Ans :

A×A = {-1,0}×{-1,0} = {(-1,-1), (-1,0), (0,-1), (0,0)}

A×A×A = {(-1,-1), (-1,0), (0,-1), (0,0)} ×{-1,0}

= {(-1,-1,-1), (-1,0,-1), (0,-1,-1), (0,0,-1), (-1,-1,1), (-1,0,1), (0,-1,1), (0,0,1)}

Q. A = {1,2,3}, B = {3,4}, C = {4,5}, Verify that A×(B∩C)= (A×B)∩(A×C)

Ans :

L H S

B∩C = {4}

A×(B∩C) = {1,2,3}×{4}

= {(1,4), (2,4), (3,4)}

R H S

A×B = {1,2,3}×{3,4}

= {(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)}

A×C = {1,2,3}×{4,5}

= {(1,4), (1,4),(2,4),(2,5),(3,4),(3,5)}

(A×B)∩(A×C) = {(1,4),(2,4),(3,4)}

L H S = R H S

Q. A = {1,2,3,…14}. R is a relation on A defined by

R = {(x,y) : 3x-y = 0}. Write R in roster form. Also write its domain, co domain and range?

Ans :

3x-y = 0

Put x = 1

3×1 – y = 0

3 – y = 0

Y = 3

Put x = 2, we get y = 6

Put x = 3, we get y = 9
Put x = 4, we get y = 12
Put x = 5, we get y = 15

R = {(1,3),(2,6),(3,9),(4,12)}

Domain = {1,2,3,4}

Range = {3,6,9,12}

Co domain = A

Q. Let A = {x,y,z}, B = {1,2} find the number of relations from A to B?

Ans:

m = 3, n= 2

Number of relations = 2^{mn}

$= 2^{3\times 2} = 2^6 = 64$

Q. A = {1,2,3}, B = {4,6,9}. R is a relation from A to B defined by R = {(x,y) : the difference between x and y is odd}. Write R in roster form. Also write its domain, co domain and range?

Ans :

A×B = {(1,4), (1,6), (1,9), (2,4), (2,6), (2,9), (3,4),(3,6), (3,9)}

R = {(1,4), (1,6), (2,9), (3,4), (3,6)}

Domain = {1,2,3}

Range = {4,6,9}

Co domain = B

Q. A×A has 9 elements of which two elements are (-3,0) and (0,3). R is a Relation on A defined by R = {(x,y) : x + y = 0}. Write r in roster form

Ans :

A×A has 9 elements

A has 3 elements

A = {-3,0,3}

R = {(-3,3), (3,-3), (0,0)}

Q. A = {1,2,3,4}. R is a relation on A defined by R = {(a,b) : b is exactly divisible by a}. Write R in roster form

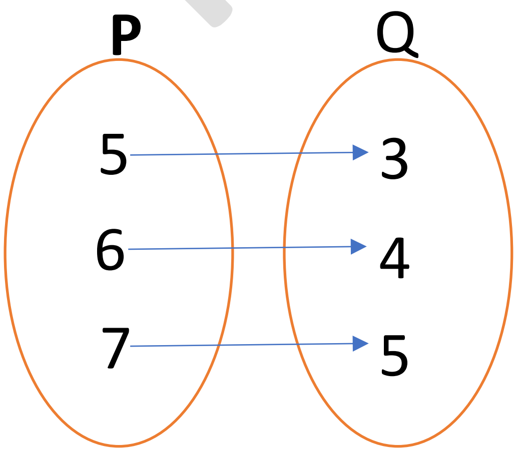
Ans :

A×A = {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)}

R = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4) }

Q. The figure shows a relation from P to Q.

Write the relation in roster form and set builder form



Ans :

Roster form

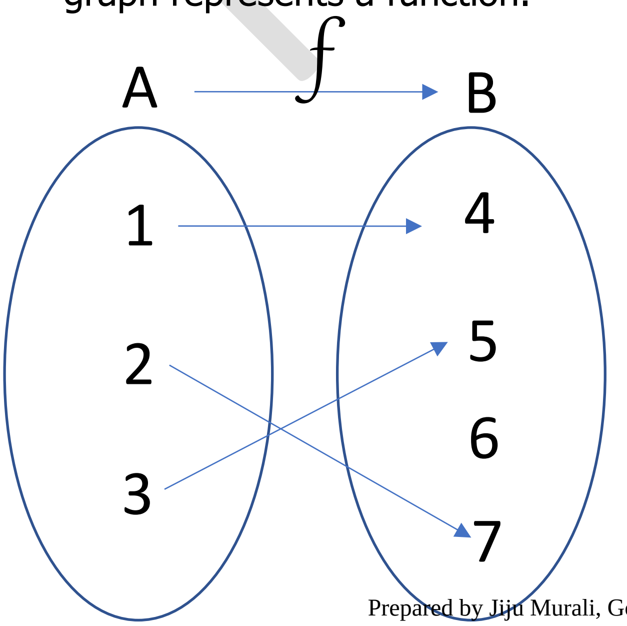
{(5,3), (6,4), (7,5)}

Set builder form

R = {(x,y) : x-y = 2, x$\in$P,y$\in$Q}

Functions

1. A relation from A to B is called a function if every element of A is related to unique element of B
2. Functions are generally denoted by f,g,h…
3. f(x) = y means y is the image of x or x is the pre image of y
4. Consider f : A $\longrightarrow$ B . A is called domain and B is called co domain
5. The set of elements in the co domain which have pre image is called range. Range is a subset of co domain
6. Number of functions from A to B is $n(B)^{n(A)}$
7. If every vertical line meets a graph only at a point, the graph represents a function.



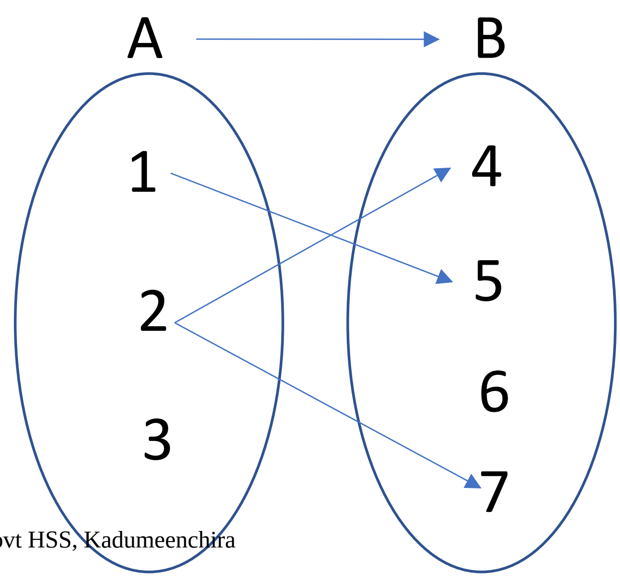


Fig 1 represents a function since every element in A has unique image in B

Fig 2 is not a function since 2 has more than one image and 3 has no image

Q. Find the domain and range of f(x) = $\frac{x^2+1}{x^2-5x+6}$

$x^2 - 5x + 6 = 0$

a =1, b = -5, c = 6

x = $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ = 2,3

Domain = R – {2,3}

Q. Find the domain and range of f(x) = $\frac{x+1}{x-1}$

x – 1 = 0

x = 1

Domain = R – {1}

f(x) = y

$\frac{x+1}{x-1}$ = y

x+1 = y(x-1)

x+1 = xy – y

x-xy = -y -1

x(1-y) = -y-1

x = $\frac{-y-1}{1-y}$

1-y = 0 , y = 1

Range = R – {1}

Q. f(x) = x^2, g(x) = 2x + 1, find f+g, f-g, fg and $\frac{f}{g}$

f+g(x) = f(x)+g(x) = $x^2 + 2x + 1$
f-g(x) = f(x) – g(x) = x^2-(2 x +1)
fg(x) = f(x)g(x) = x^2(2 x +1)
$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x^2}{2x+1}$

Q. Find the domain and range of f(x) = $\sqrt{x+1}$

$x + 1 \geq 0$

$x \geq -1$

Domain = [-1, ∞)

Range = [0, ∞)

Q. Find the domain and range of f(x) = $\sqrt{16 - x^2}$

Domain = [-4,4]

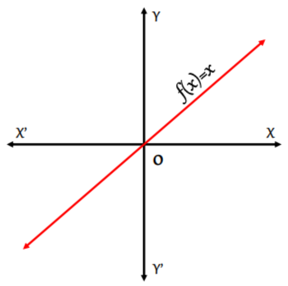
Range = [0,4]

Identity function

$\boxed{f(x) = x}$

Domain = R

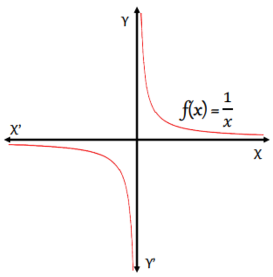
Range = R



$$\boxed{f(x)=\frac{1}{x}}$$

Domain = R – {o}

Range = R – {o}

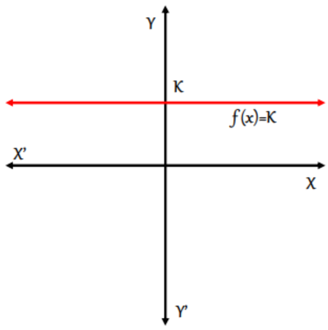


Constant function

$$\boxed{f(x)=k}$$

Domain = R

Range = {k}



Graph of a constant function is a straight line parallel to x axis

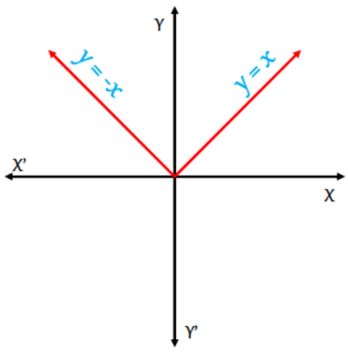
Modulus function

$$\boxed{f(x)=|x|}$$

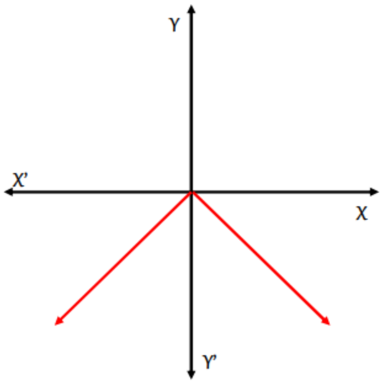
$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Domain = R

Range = $[0,\infty)$



$f(x) = -|x|$



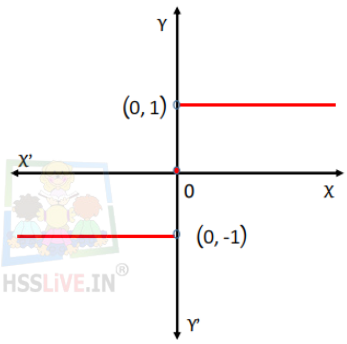
Domain = R

Range = $(-\infty, 0]$

Signum function

$$f(x) = \begin{cases} \frac{|x|}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$$

$$f(x) = \begin{cases} -1, x < 0 \\ 0, x = 0 \\ 1, x > 0 \end{cases}$$

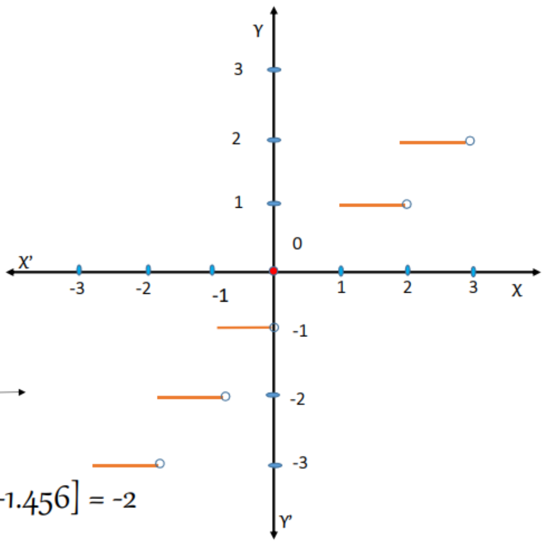


Domain = R

Range = {-1, 0, 1}

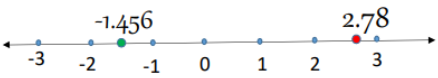
Greatest integer function

$f(x) = [x]$



Domain = R

Range = Z



$[4] = 4$ $[-2] = -2$ $[2.78] = 2$ $[-1.456] = -2$

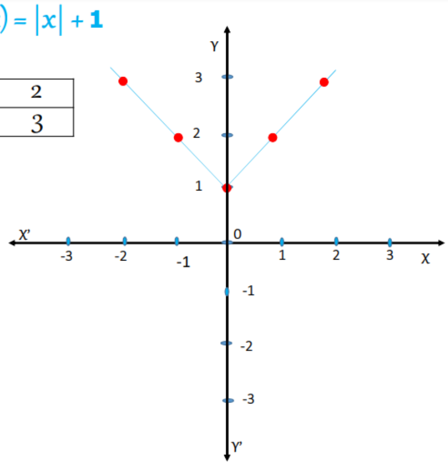
Q. Draw the graph of the function $f(x) = |x| + 1$

x	-2	-1	0	1	2
y	3	2	1	2	3

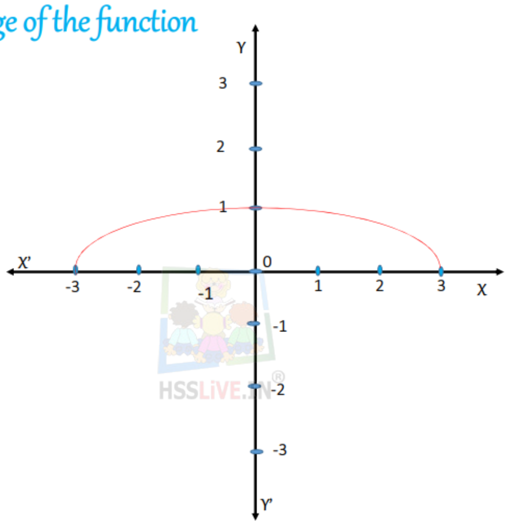
$|-2| + 1$
$= 2 + 1$
$= 3$

Domain = R

Range = $[1, \infty)$



Q. Write the domain and range of the function



Domain = $[-3, 3]$

Range = $[0, 1]$

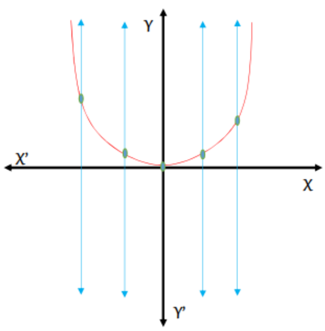


Fig 1

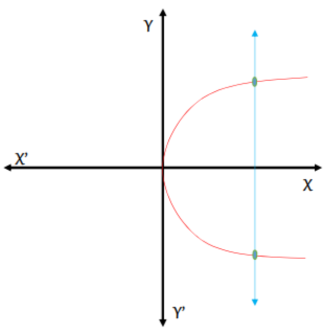
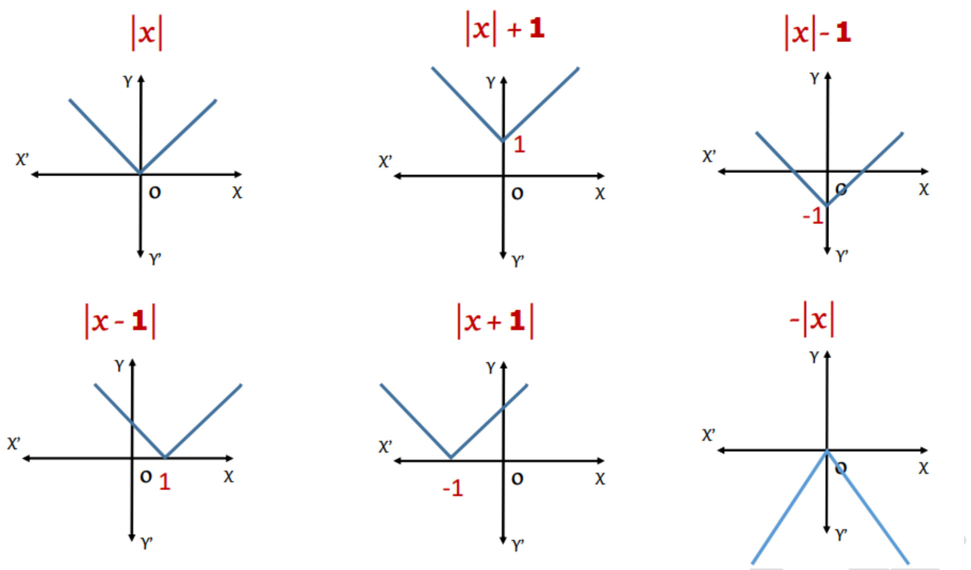


Fig 2

Vertical line test : Every vertical line on the graph of a function meets the graph only at one point. Fig 1 represents a function. Fig 2 is not a function.



Chapter 3

Trigonometry

<u>Units of measurement of angles</u>

1. Degree

If the rotation from initial side to the terminal side is $(\frac{1}{360})^{th}$ of a revolution, the angle is said to have a measure of one degree (1^0)

1^0 = 60 minutes = $60^{/}$

$1^{/}$ = 60 seconds = $60^{//}$

2. Radian

One radian (1^c) is the angle subtended at the centre of a circle by an arc same to its radius

Degree $\times\frac{\pi}{180}$ = radian

Radian $\times\frac{180}{\pi}$ = degree

Degree	30	45	60	90	180	270	360
Radian	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

1. Convert 240^0 into radian

   $240 \times\frac{\pi}{180} = \frac{4\pi}{3}$

2. Convert $40^0 20^/$ into radian

   $40^0 + 20^/$

   $= 40^0 + \frac{20}{60}^{\,0}$

   $= 40^0 + \frac{1}{3}^{\,0}$

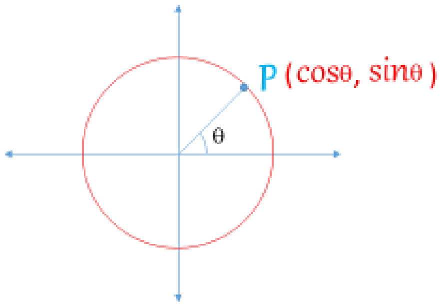
   $= \frac{120+1}{3}^{\,0}$

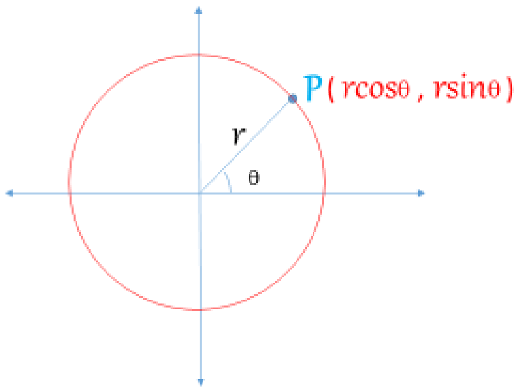
   $=\frac{121}{3}^{\,0}$

   $= \frac{121}{3}\times\frac{\pi}{180} = \frac{121\pi}{540}$

3. Convert $\frac{4\pi}{3}$ radian into degree

   $\frac{4\pi}{3}\times\frac{180}{\pi} = 240^0$





$\frac{sin\theta}{cos\theta}$ = tanθ $\frac{1}{sin\theta}$ = cosecθ $\frac{1}{cos\theta}$ = secθ $\frac{1}{tan\theta}$ = cotθ

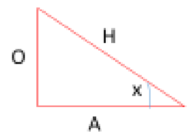
$\frac{cos\theta}{sin\theta}$ = cotθ $\frac{1}{cosec\theta}$ = sinθ $\frac{1}{sec\theta}$ = cosθ $\frac{1}{cot\theta}$ = tanθ

Trigonometric functions

O : opposite side O $=\sqrt{H^2 - A^2}$

A : adjacent side A $=\sqrt{H^2 - O^2}$

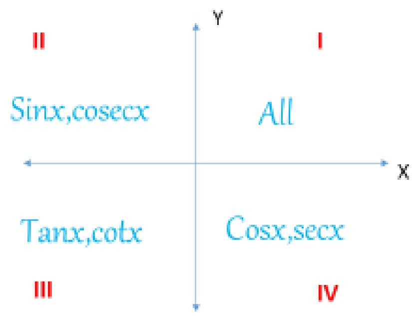
H : hypotenuse H $=\sqrt{A^2 + O^2}$



Sinx = $\frac{O}{H}$ cosecx = $\frac{H}{O}$

cosx = $\frac{A}{H}$ secx = $\frac{H}{A}$

tanx = $\frac{O}{A}$ cotx = $\frac{A}{O}$



Q. tanx = - $\frac{5}{12}$, x lies in second quadrant, find the values of other five trigonometric functions

$$\text{tanx} = \frac{5}{12} = \frac{O}{A}$$

O =5 and A = 12

$$\text{H} = \sqrt{O^2 + A^2}$$

$$= \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169} = 13$$

$$\text{sinx} = \frac{O}{H} = \frac{5}{13}$$

$$\text{cosx} = \frac{A}{H} = \frac{-12}{13}$$

$$\text{tanx} = \frac{O}{A} = \frac{-5}{12}$$

$$\text{cosecx} = \frac{H}{O} = \frac{13}{5}$$

$$\text{secx} = \frac{H}{A} = -\frac{13}{12}$$

$$\text{cotx} = \frac{A}{O} = -\frac{12}{5}$$

We can do this type of problems using the formulas listed below

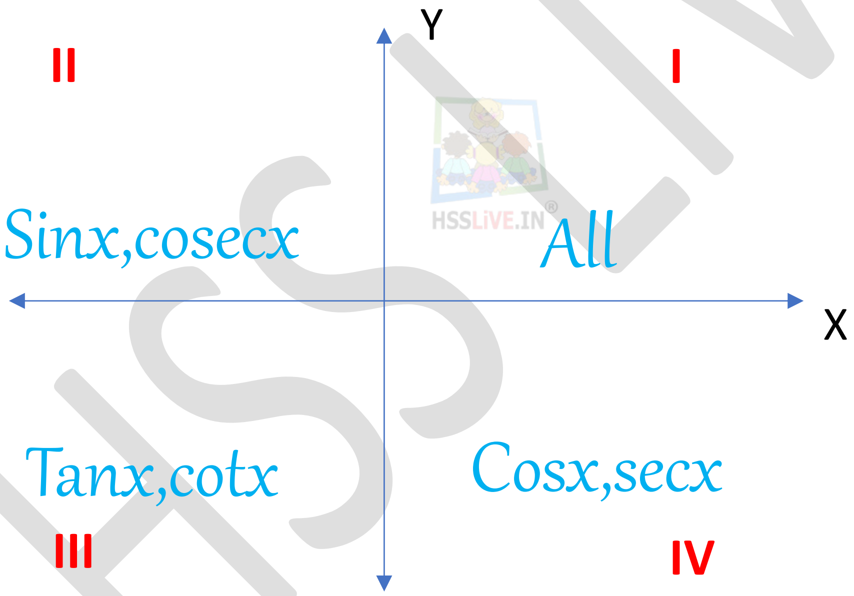
$\sin^2 x + \cos^2 x = 1$

$\sec^2 x - \tan^2 x = 1$

$\text{cosec}^2 x - \cot^2 x = 1$

	0°	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

In first quadrant, all functions are positive. In second quadrant sinx and cosecx are positive. In third quadrant, tanx and cotx are positive. In fourth quadrant, cosx and secx are positive



Q. Prove that $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

LHS

$2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$= 2\sin^2 135^0 + 2\cos^2 45^0 + 2\sec^2 60^0$

$Sin135^0$

$= \sin(90+45)$

$=\cos45^0$

$= \frac{1}{\sqrt{2}}$

$= 2(\frac{1}{\sqrt{2}})^2 + 2(\frac{1}{\sqrt{2}})^2 + 2(2)^2$

$=2\times\frac{1}{2} + 2\times\frac{1}{2} + 2\times4 = 10$

$Sin(-x) = - sinx$

$Cos(-x) = cosx$

$Tan(-x)= -tanx$

$Cot(-x) = -cotx$

$Sec(-x) = secx$

$Cosec(-x) = -cosecx$

$Sin(x+y) = sinx\ cosy + cosx\ siny$

$Sin(x-y) = sinx\ cosy - cosx\ siny$

$cos(x+y) = cosx\ cosy - sinx\ siny$

$cos(x-y) = cosx\ cosy + sinx\ siny$

tan(x+y)= $\frac{tanx+tany}{1-tanxtany}$

tan(x-y)= $\frac{tanx-tany}{1+tanxtany}$

cot(x+y) = $\frac{cotxcoty-1}{coty+cotx}$

cot(x-y) = $\frac{cotxcoty+1}{coty-cotx}$

Q. sin 75^0

= sin (45^0+30^0)

= $\sin45^0 \cos30^0 + \cos45^0 \sin30^0$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

Q. tan 75^0

= $\tan(45^0+30^0)$

$= \frac{tan45^0+tan30^0}{1-tan45^0 tan30^0}$

$= \frac{1+\frac{1}{\sqrt{3}}}{1-1.\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

Q. Prove that $\frac{\sin(x+y)}{\sin(x-y)} = \frac{tanx+tany}{tanx-tany}$

R H S

$$\frac{tanx+tany}{tanx-tany} = \frac{\frac{sinx}{cosx}+\frac{siny}{cosy}}{\frac{sinx}{cosx}-\frac{siny}{cosy}}$$

$$= \frac{\frac{sinxcosy+cosxsiny}{cosxcosy}}{\frac{sinxcosy-cosxsiny}{cosxcosy}}$$

$$= \frac{sinxcosy+cosxsiny}{sinxcosy-cosxsiny} = \frac{\sin(x+y)}{\sin(x-y)} = \text{LHS}$$

Q. Prove that $\frac{\tan(\frac{\pi}{4}+x)}{\tan(\frac{\pi}{4}-x)} = (\frac{1+tanx}{1-tanx})^2$

LHS

$$\frac{\tan(\frac{\pi}{4}+x)}{\tan(\frac{\pi}{4}-x)} = \frac{tan\frac{\pi}{4}+tanx}{1-tan\frac{\pi}{4}\,tanx} \div \frac{tan\frac{\pi}{4}-tanx}{1+tan\frac{\pi}{4}\,tanx}$$

$$= \frac{1+tanx}{1-tanx} \div \frac{1-tanx}{1+tanx}$$

$$= \frac{1+tanx}{1-tanx} \times \frac{1+tanx}{1-tanx}$$

$$= (\frac{1+tanx}{1-tanx})^2 = \text{RHS}$$

Q. Prove that tan3A – tan2A – tanA = tan3A tan2A tanA

3A = 2A + A

tan3A = tan(2A + A)

tan3A = $\frac{tan2A+tanA}{1-tan2A\,tanA}$

tan3A(1-tan2A tanA) = tan2A + tanA

tan3A – tan3A tan2A tanA = tan2A + tanA

tan3A – tan2A – tanA = tan3A tan2A tanA

Sinx + siny = 2 sin$\frac{x+y}{2}$ cos$\frac{x-y}{2}$

Sinx - siny = 2 cos$\frac{x+y}{2}$ sin$\frac{x-y}{2}$

cosx + cosy = 2 cos$\frac{x+y}{2}$ cos$\frac{x-y}{2}$

cosx - cosy = - 2 sin$\frac{x+y}{2}$ sin$\frac{x-y}{2}$-

Q. prove that $\frac{sin5x+sin3x}{cos5x+cos3x}$ = tan4x

$\frac{sin5x+sin3x}{cos5x+cos3x} = \frac{2\ sin\frac{5x+3x}{2}\ cos\frac{5x-3x}{2}}{2\ cos\frac{5x+3x}{2}\ cos\frac{5x-3x}{2}}$

$= \frac{sin4x\ cosx}{cos4x\ cosx} = \frac{sin4x}{cos4x}$ = tan4x

Q. Prove that $(cosx + cosy)^2 + (sinx - siny)^2 = 4\cos^2\frac{x+y}{2}$

LHS

$(cosx + cosy)^2 + (sinx - siny)^2 = (2\cos\frac{x+y}{2}\cos\frac{x-y}{2})^2 + (2\cos\frac{x+y}{2}\sin\frac{x-y}{2})^2$

$= 4\cos^2\frac{x+y}{2}\cos^2\frac{x-y}{2} + 4\cos^2\frac{x+y}{2}\sin^2\frac{x-y}{2}$

$= 4\cos^2\frac{x+y}{2}(\cos^2\frac{x-y}{2} + \sin^2\frac{x-y}{2})$

$= 4\cos^2\frac{x+y}{2} \times 1 = 4\cos^2\frac{x+y}{2}$ = R H S

Q. Prove that $\frac{cos4x+cos3x+cos2x}{sin4x+sin3x+sin2x}$ = cot3x

$\frac{cos4x+cos3x+cos2x}{sin4x+sin3x+sin2x}$

$=\frac{(cos4x+cos2x)+cos3x}{(sin4x+sin2x)+sin3x}$

$= \frac{2cos3xcosx+cos3x}{2sin3xcosx+sin3x}$

$= \frac{cos3x(2cosx+1)}{sin3x(2cosx+1)}$

$= \frac{cos3x}{sin3x}$

= cot3x

= R H S

Chapter 4

Mathematical Induction

Statement

A sentence which can be judged to be true or false is called a statement

Mathematical statement

statement involving mathematical relations are called mathematical statements

Principle of mathematical induction

Let P(n) be a mathematical statement involving the natural number n

Step 1

Prove that P(1) is true

Step 2

Assume that P(k) is true

Step 3

Prove that P(k+1) is true

Q. 1.2 + 2.3 + 3.4 + …+ n(n+1) = $\frac{n(n+1)(n+2)}{3}$

Let P(n) = 1.2 + 2.3 + 3.4 + …+ n(n+1) = $\frac{n(n+1)(n+2)}{3}$

P(1): 1.2 = $\frac{1(1+1)(1+2)}{3}$

2 = $\frac{2.3}{3}$

2 = 2

P(1) is true

P(2): 1.2 + 2.3 = $\frac{2(2+1)(2+2)}{3}$

8 = $\frac{2.3.4}{3}$

8 = 8

P(2) is true

P(k): 1.2 + 2.3 + 3.4 + …+ k(k+1) = $\frac{k(k+1)(k+2)}{3}$

Assume that p(k) is true

P(k+1): 1.2 + 2.3 + 3.4 + …+ (k+1)(k+2) = $\frac{(k+1)(k+1+1)(k+1+2)}{3}$ = $\frac{(k+1)(k+2)(k+3)}{3}$

We have to prove that p(k+1) is true

L.H.S

1.2 + 2.3 + 3.4 + …+ (k+1)(k+2)

=[1.2 + 2.3 + 3.4 + ...+ k(k+1)]+ (k+1)(k+2)

= $\frac{k(k+1)(k+2)}{3}$ + (k+1)(k+2)

= (k+1)(k+2) [$\frac{k}{3}$ + 1]

= $\frac{(k+1)(k+2)(k+3)}{3}$

= R.H.S

P(k+1) is true.

Hence by P.M.I then result is true for all natural number n

Q. 1+ 3+ 3^2 + ... + 3^{n-1} = $\frac{3^n-1}{2}$

P(n) : 1+ 3+ 3^2 + ... + 3^{n-1} = $\frac{3^n-1}{2}$

P(1): 1 = $\frac{3^1-1}{2}$

1 = 1

P(1) is true

P(k) : 1+ 3+ 3^2 + ... + 3^{k-1} = $\frac{3^k-1}{2}$

Assume that p(k) is true

P(k+1) : 1+ 3+ 3^2 + ... + 3^{k+1-1} = $\frac{3^{k+1}-1}{2}$

We have to prove that p(k+1) is true

L.H.S

1+ 3+ 3^2 + ... + 3^k

= [1+ 3+ 3^2 + ...+ 3^{k-1}] + 3^k

$= \frac{3^k - 1}{2} + 3^k$

$= \frac{3^k - 1 + 2.3^k}{2}$

$= \frac{3.3^k - 1}{2} = \frac{3^{k+1} - 1}{2}$ = R H S

P(k+1) is true.

Hence by P.M.I then result is true for all natural number n

Q. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

P(n): $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

P(1) : $\frac{1}{2} = 1 - \frac{1}{2^1}$

$\frac{1}{2} = \frac{1}{2}$

P(1) is true

P(k) : $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$

Assume that p(k) is true

P(k+1): $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^{k+1}}$

We have to prove that p(k+1) is true

LHS

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^{k+1}}$

$=[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^k}] + \frac{1}{2^{k+1}}$

$= 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$

$= 1 - \frac{1}{2^k} + \frac{1}{2^k.2^1}$

$=1 - \frac{1}{2^k}\ (1 - \frac{1}{2})$

$= 1 - \frac{1}{2^k} . \frac{1}{2}$

$= 1 - \frac{1}{2^{k+1}}$ = RHS

P(k+1) is true.

Hence by P.M.I then result is true for all natural number n

Chapter 5.

Complex Numbers

Imaginary number

An imaginary number is the square root of a negative real number. It can be written as a real number multiplied by the imaginary unit **i** (iota)

$\sqrt{-1}$ = i

$i^2 = -1 \quad i^3 = -i \quad i^4 = 1$

$i^0 = i^4 = i^8 = i^{12} = \ldots = 1 \quad i^{4n} = 1$

$\sqrt{-9} = \sqrt{9 \times -1} = \sqrt{9} \times \sqrt{-1} = 3i$

$\sqrt{-4} = 2i$

$\sqrt{-5} = i\sqrt{5}$

i^n can be written as i^k, where k is the remainder obtained by dividing n by 4

$i^{10} = i^2 = -1$ (2 is the remainder when 10 is divided by 2)

$i^{51} = i^3 = -i$

$i^{-39} = \frac{1}{i^{39}} = \frac{1}{i^3} = \frac{i^4}{i^3} = i$

$i^9 + i^{19} = i^1 + i^3 = i - i = 0$

Complex Number

A number of the form a+ib, where a and b are real numbers is called a complex number. Complex numbers are generally denoted by z

A is called real part(Rez) and b is called imaginary part(Imz)

Eg : $z = -2 + \sqrt{3}\, i$ Rez = -2, Imz = $\sqrt{3}$

Every real number is a complex number

7 = 7 + 0i

Set of all complex numbers is denoted by C. R is a subset of C

A complex number with real part 0 is called purely imaginary complex number

Eg: i,3i,-i,-2i

If two complex numbers are equal, their real parts are equal and imaginary parts are equal

Q. If 4x+i(3x-y) = 8-6i, find x and y

Equating the real part

4x = 8   x = 8/4 = 2

Equating the imaginary part

3x-y = -6

3×2 – y = -6

6 – y = -6

Y = 6 + 6 = 12

Conjugate of a complex number

Conjugate of z = a+ib is denoted by $\bar{z}$ and it is defined as $\bar{z}$ = a-ib

Eg: z = -1-i, $\bar{z}$ = -1+i

Modulus of a complex number

Modulus of z = a+ib is denoted by |z| and is defined as

$|z| = \sqrt{a^2 + b^2}$

Eg : z = 3-4i

$|z| = \sqrt{3^2 + -4^2}$

$= \sqrt{9 + 16} = 5$

Q. Find the sum $i + i^2 + i^3 + ... + i^{100}$

Ans:

Sequence is a G.P with a = i, r = i

$S_n = \frac{a(r^n-1)}{r-1}$

$S_{100} = \frac{i(i^{100}-1)}{i-1}$

$= \frac{i(1-1)}{i-1} = 0$

Q. $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9}$

Ans:

= 5i + 3×2i + 2×3i

= 5i + 6i + 6i

= 17i

Operations

(2+3i) + (-1+2i) = (2+ -1) + (3i+2i) = 1 + 5i

(1-i) – (4-2i) = 1 –i -4+2i = -3-i

$(2+i)^2 = 2^2 + 2.2.i + i^2 = 4 + 4i - 1 = 3 + 4i$

$(2+3i)(3-2i) = 6-4i+9i-6i^2 = 6+5i-6\times-1 = 6+5i+6 = 12 + 5i$

Negative real numbers has no real roots

Q. write $\frac{2+3i}{4+5i}$ in a+ib form

Ans:

$\frac{(2+3i)(4-5i)}{(4+5i)(4-5i)} =$

$$\frac{8-10i+12i+15}{(4)^2-(5i)^2}$$

$$\frac{23+2i}{16+25}$$

$\frac{23+2i}{41} = \frac{23}{41} + \frac{2\,i}{41}$

Q. $(1+i)^4$

$(1+i)^2 = 1^2 + 2\times1\times i + i^2$

$= 1+2i-1 = 2i$

$(1+i)^4 = [(1+i)^2]^2 = (2i)^2 = 2^2\times i^2 = 4\times-1 = -4 = -4+0i$

Q. $\frac{3-4i}{(4-2i)(1+i)}$

$= \frac{3-4i}{4+4i-2i+2}$

$= \frac{3-4i}{6+2i}$

$= \frac{(3-4i)(6-2i)}{(6+2i)(6-2i)}$

$= \frac{18-6i-24i-8}{6^2-(2i)^2} = \frac{10-30i}{36+4} = \frac{10-30i}{40}$

= $\frac{10}{40}$ - $\frac{30i}{40}$ = $\frac{1}{4}$ - $\frac{3i}{4}$

Q. Find the least positive value of n such that $(\frac{1+i}{1-i})^n = 1$

$$\frac{1+i}{1-i}$$

= $\frac{(1+i)(1+i)}{(1-i)(1+i)}$

= $\frac{1+i+i+i^2}{1^2-i^2}$

$$= \frac{1+2i-1}{1+1}$$

$$= \frac{2i}{2}$$

= i

$(\frac{1+i}{1-i})^n = 1$

$i^n = 1$

$i^4 = 1$

n = 4

Q. Find the multiplicative inverse of 4 + 3i

$\frac{1}{4+3i}$ = $\frac{(4-3i)}{(4+3i)(4-3i)}$

= $\frac{(4-3i)}{4^2-(3i)^2}$

= $\frac{4-3i}{16+9}$

= $\frac{4-3i}{25}$

$= \frac{4}{25} - \frac{3i}{25}$

Chapter 6

Linear Inequalities

Two real numbers or two algebraic expressions related by the symbol

$<$, $\leq$, $>$,$\geq$ form an inequality.

When both sides of an inequality are multiplied or divided by the same negative number, then the sign of inequality is reversed.

Q. Solve $2(x+3)-10 \leq 6(x-2)$ for real x. represent the solution on a number line

$2(2x+3)-10 \leq 6(x-2)$

$4x+6-10 \leq 6x-12$

$4x- 4 \leq 6x - 12$

$4x-6x \leq -12 + 4$

$-2x \leq -8$

$X \geq \frac{-8}{-2}$

$X \geq 4$

$x \in [4,\infty)$

Q. solve $\frac{3(x-2)}{5} > \frac{5(2-x)}{3}$

$9(x-2) > 25(2-x)$

$9x - 18 > 50 -25x$

$9x + 25x > 50 + 18$

34x > 68

X > $\frac{68}{34}$

X > 2

x∈(2,∞)

Q. solve 7 ≤$\frac{(3x+11)}{2}$≤ 11

7×2 ≤ $\frac{(3x+11)}{2}$ × 2 ≤ 11×2

14 ≤3x+11≤22

14-11 ≤ 3x ≤ 22-11

3≤3x≤11

1 ≤ x ≤$\frac{11}{3}$

x∈[1,$\frac{11}{3}$]

Q. Ravi obtained 70 and 75 marks in first two unit tests. Find the minimum marks he should get in the third test to have an average of at least 60 marks

Let x be the marks in the third test.

Average = $\frac{70+75+x}{3}$

= $\frac{145+x}{3}$

Average ≥60

$\frac{145+x}{3}$≥ 60

145 + x ≥180

x≥180 -145

x ≥35

Minimum marks in the third test = 35

Solve graphically

$3x+4y \le 12$, $4x+3y \le 12$, $x \le 2$, $x \ge 0$, $y \ge 0$

$3x+4y \le 12$

$3x+4y = 12$

x	0	4
y	3	0

$3x+4y \le 12$

$3.0+4.0 \le 12$

$0 \le 12$

True

$4x+3y \le 12$

$4x+3y = 12$

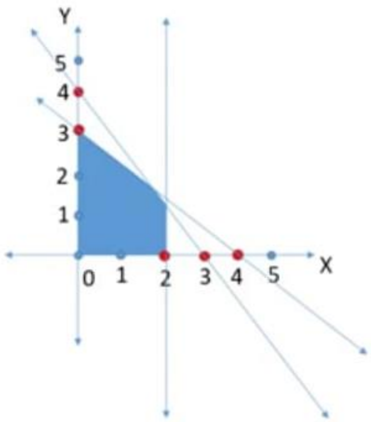
x	0	3
y	4	0

$4x+3y \le 12$

$4.0+3.0 \le 12$

$0 \le 12$

True



$x \le 2$

$x = 2$

Chapter 7

Permutations and Combinations

Fundamental principle of counting

If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrences of the events in the given order is m×n.

1. How many three digit numbers can be formed using the digits 1,2,3,4,5 if
   a) repetition of the digits is allowed
   b) repetition of the digits is not allowed

   a) $5\times5\times5 = 125$
   b) $5\times4\times3 = 60$
2. How many three digit even numbers can be formed using the digits 1,2,3,4,5,6 if

   a) repetition of the digits is allowed

   b) repetition of the digits is not allowed

a) $6 \times6 \times3 =108$

b) 4 ×5 ×3= 60

3. How many 5 digit telephone numbers can be constructed if each number starts with 67 and no digit is repeated

8 ×7 ×6 = 336

4. How many 4 digit numbers are there with no digit repeated

9 ×9 ×8 ×7 = 4536

Factorial

The continued product of first n natural numbers is called n factorial and is denoted by n!

n! = 1 ×2 ×3 ×….(n-1) ×n

0! =1

1! =1

2! = 1 ×2 =2

3! = 1 ×2 ×3 = 6

4! =1 ×2 ×3 ×4 = 24

5! = 1 ×2 ×3 ×4 ×5
= 5 ×4!
= 5 ×4 ×3!

10! = 10 ×9!
= 10 ×9 ×8!

n! =n(n-1)!
= n(n-1)(n-2)!

1. Find 4! + 3!
   = 24+6 = 30

2. Find $\frac{10!}{8!}$
   $\frac{10!}{8!} = \frac{10\times9\times8!}{8!}$ = 90

3. $\frac{n!}{(n-1)!}$

$\frac{n!}{(n-1)!} = \frac{n(n-1)!}{(n-1)!} = n$

4. $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$ Find x

$\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$

$\frac{1}{6!} + \frac{1}{7\times6!} = \frac{x}{8\times7\times6!}$

$\frac{1}{1} + \frac{1}{7} = \frac{x}{8\times7}$

$\frac{8}{7} = \frac{x}{8\times7}$

$x = 64$

<u>Combination</u>

Combination is a mathematical method to find number of selections.

^nC_r is the total number of selections out of n objects taking r at a time(r≤n).

$$^nC_r = \frac{n!}{r!(n-r)!}$$

$^nC_1 = n$

$^nC_2 = \frac{n(n-1)}{1\times2}$

$^nC_3 = \frac{n(n-1)(n-2)}{1\times2\times3}$

$^nC_n = 1$

$^{10}C_1 = 10$

$^{10}C_2 = \frac{10\times9}{1\times2}$

$^{10}C_3 = \frac{10\times9\times8}{1\times2\times3}$

$$^nC_r = {}^nC_{n-r}$$

$^{10}C_7 = {}^{10}C_{10-7} = {}^{10}C_3$

$^{100}C_{98} = {}^{100}C_{100-98} = {}^{100}C_2$

Q. $^nC_8 = {}^nC_9$. Find $^nC_2$

n = 8+9 = 17.

$^nC_2 = {}^{17}C_2$

$= \frac{17\times16}{1\times2} = 136$

Q. How many chords can be drawn through 21 points on a circle

A chord is formed by joining any two points on a circle.To form a chord we need 2 points. There are 21 points

Number of chords = $^{21}C_2 = \frac{21\times20}{1\times2} = 210$

Q. Find the number of diagonals of an octagon

Number of diagonals of a polygon = $^nC_2 - n$

Number of diagonals of an octagon = $^8C_2 - 8 = 28 - 8 = 20$

Q. In how many ways can 2 boys and 3 girls be selected from 4 boys and 5 girls

$^4C_2 \times {}^5C_3 = \frac{4\times3}{1\times2} \times \frac{5\times4\times3}{1\times2\times3} = 6\times10 = 60$

Q. In how many ways can a cricket team of 11 be selected from 17 in which 5 players can bowl and each team of 11 must contain 4 bowlers

12 batsmen 5 bowlers

7 batsmen 4 bowlers

$^{12}C_7 \times {}^5C_4 = 3960$

Q. The English alphabet has 5 vowels and 21 consonants. How many words with 2 different vowels and 2 different consonants can be formed?

Total number of selections = $^5C_2 \times {}^{21}C_2 = 2100$

Using each selection of 4 letters, 4! Words can be formed

Total number of words = $2100 \times 4! = 5040$

Q. How many words with or without meaning each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER

3 vowels 5 consonants

2 vowels 3 consonants

Total number of selections = $^3C_2 \times {}^5C_3 = 30$

Using each selection of 5 letters, 5! Words can be formed

Total words = $30 \times 5! = 3600$

Q. In an examination, a question paper consists of 12 questions divided into two parts part I and part II containing 5 and 7 questions respectively. A student is required to attempt 8 questions in all selecting at least 3 from each part. In how many ways can a student select the questions?

Part I 5	Part II 7
3	5
4	4
5	3

Total = $^5C_3.{}^7C_5 + {}^5C_4.{}^7C_4 + {}^5C_5.{}^7C_3 = 420.$

Q. ${}^{2n}C_3 : {}^{n}C_3 = 12 : 1$ find n

$$\frac{2n_{C3}}{n_{C3}} = \frac{12}{1}$$

$$\frac{\frac{2n(2n-1)(2n-2)}{1\times2\times3}}{\frac{n(n-1)(n-2)}{1\times2\times3}} = \frac{12}{1}$$

$$\frac{2(2n-1)2(n-1)}{(n-1)(n-2)} = 12$$

$$\frac{2n-1}{n-2} = 3$$

2n-1 = 3(n-2)

2n-1= 3n-6

3n-2n = -1+6

n= 5

Q. In a school there are 20 teachers. In how many ways can a principal and vice principal can be selected

The principal can be selected in 20 ways and vice principal can be selected in 19 ways.

Total = 20×19 = 380

Chapter 8

Binomial Theorem

Binomial theorem for any positive integer n

$$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1}\, b + {}^nC_2\, a^{n-2}\, b^2 + \ldots + {}^nC_n\, b^n$$

$$= a^n + {}^nC_1\, a^{n-1}\, b + {}^nC_2\, a^{n-2}\, b^2 + \dots + b^n$$

Q. Expand $(x^2 + \frac{3}{x})^4$

$(x^2 + \frac{3}{x})^4 = (x^2)^4 + {}^4C_1\,(x^2)^3\,(\frac{3}{x}) + {}^4C_2\,(x^2)^2\,(\frac{3}{x})^2 + {}^4C_3\,(x^2)\,(\frac{3}{x})^3 + (\frac{3}{x})^4$

$= x^8 + 4.x^6.(\frac{3}{x}) + 6.x^4.\frac{9}{x^2} + 4.x^2.\frac{27}{x^3} + \frac{81}{x^4}$

$= x^8 + 12x^5 + 54x^2 + \frac{108}{x} + \frac{81}{x^4}$

$(x^2 - \frac{3}{x})^4 = x^8 - 12x^5 + 54x^2 - \frac{108}{x} + \frac{81}{x^4}$

Q. $(97)^3$

$(97)^3 = (100-3)^3$

$= 100^3 - {}^3C_1(100)^2.3 + {}^3C_2(100)(3)^2 - 3^3$

$= 1000000 - 3.10000.3 + 3.\ 100.9 - 27$

$= 1000000 - 90000 + 2700 - 27$

$= 912673$

Q. Find $(a+b)^4 - (a-b)^4$. Hence evaluate $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$

$(a+b)^4 = a^4 + {}^4C_1\, a^3\, b + {}^4C_2\, a^2\, b^2 + {}^4C_3\, a\, b^3 + b^4$

$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

$(a+b)^4 - (a-b)^4 = 8a^3b + 8ab^3$

$= 8ab(a^2+b^2)$

$(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 = 8\sqrt{3}\,\sqrt{2}\,(\sqrt{3}^2 + \sqrt{2}^2)$

$= 8\sqrt{6}\,(3+2) = 40\sqrt{6}$

Note:

1. The number of terms in the expansion of $(a+b)^n$ is n+1
2. $T_1, T_2, T_3, \ldots$ are the terms
3. $T_2 = {}^nC_1\, a^{n-1}\, b^1$
4. $T_3 = {}^nC_2\, a^{n-2}\, b^2$
5. $T_4 = {}^nC_3\, a^{n-3}\, b^3$
6. T_{r+1} is the general term in binomial expansion
7. $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

   Middle terms

   Let m be the number of terms in a binomial expansion.

   If m is odd it has only one middle term, $(\frac{m+1}{2})^{th}$ term.

   If m is even, it has 2 middle terms, $(\frac{m}{2})^{th}$ and $(\frac{m}{2}+1)^{th}$ terms.

Q. Find the middle term in the expansion of $(\frac{x}{3} + 9y)^{10}$

Number of terms in the expansion is 11

T_6 is the middle term

$T_6 = {}^nC_5\, a^{n-5}\, b^5$

$= {}^{10}C_5\, (\frac{x}{3})^{10-5}\, (9y)^5$

$= 252(\frac{x}{3})^5 9^5 y^5$

$= 252.\ \frac{x^5}{3^5}\ .59049.\ y^5$

$= 61236x^5y^5$

Chapter 9

Sequence and series

Geometric Progression (G.P)

A geometric progression is a sequence of non-zero numbers where each term after the first term is found by multiplying the previous one by a fixed non-zero number called common ratio.

Eg: 3,6,12,24,…

$r = \frac{a_2}{a_1}$

$a_n = a\, r^{n-1}$

$a_2 = ar$

$a_3 = ar^2$

$a_4 = ar^3$

$S_n = a\left(\frac{r^n-1}{r-1}\right)$

$S_\infty = \frac{a}{1-r}$, $|r| < 1$

Geometric mean (G.M) = $\sqrt{ab}$

Q. Which term of the sequence 2, $2\sqrt{2}$, 4, … is 128

$a = 2$

$r = \frac{a_2}{a_1} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$a_n = 128$

$a\, r^{n-1} = 128$

$2.\ \sqrt{2}^{n-1} = 128$

$\sqrt{2}^{\ n-1} = 64$

$\sqrt{2}^{\ n-1} = 2^6 = \sqrt{2}^{\ 12}$

$n-1 = 12$

$n = 13$

Q. The sum of first three terms of a G.P is 16 and the sum of next three terms is 128. Find first term and common ratio

$a_1 + a_2 + a_3 = 16$

$a + ar + ar^2 = 16$

$a(1+r+r^2) = 16$

$a_4+a_5+a_6 = 128$

$ar^3+ar^4+ar^5 = 128$

$ar^3(1+r+r^2) = 128$

$r^3 = 8$

$r = 2$

$a(1+r+r^2) = 16$

$a(1+2+2^2) =16$

$a(7) = 16$

$a= \frac{16}{7}$

Q. If the 4^{th} , 10^{th} and 16^{th} terms of a G.P are x,y,z respectively. Prove that x,y,z are in G.P

$ar^3 = x$

$ar^9 = y$

$ar^{15} = z$

$\frac{y}{x} = \frac{ar^9}{ar^3} = r^6$

$\frac{z}{y} = \frac{ar^{15}}{ar^9} = r^6$

x,y,z are in G.P

Q. Find the sum to n terms of the sequence 8,88,888,...

8 + 88 + 888 + … to n terms

= 8[1 + 11 + 111 + … to n terms]

= $\frac{8}{9}$ [9 + 99 + 999 + … to n terms]

= $\frac{8}{9}$ [(10-1) + (100-1) + (1000-1) + … to n terms]

= $\frac{8}{9}$ [(10 + 100 + 1000 + … to nerms)-(1 + 1+ 1+ … to n terms)]

= $\frac{8}{9}$ [10($\frac{10^n-1}{10-1}$) - n]

Q. The sum of three terms of a G.P is $\frac{39}{10}$ and their product is 1. Find the terms.

Let $\frac{a}{r}$, a , ar be the terms

$\frac{a}{r}$. a . ar = 1

$a^3 = 1$

a = 1

$\frac{a}{r}$ + a + ar = $\frac{39}{10}$

$\frac{1}{r}$ + 1 + r = $\frac{39}{10}$

$\frac{1}{r}$ .10r + 1. 10r + r .10r= $\frac{39}{10}$.10r

$10+10r+10r^2 = 39r$

$10r^2-29r+10 = 0$

r= $\frac{5}{2}$, $\frac{2}{5}$

$\frac{5}{2}$, 1, $\frac{2}{5}$ are the terms.

Q. A.M and G.M of two positive numbers are 10 and 8 respectively. Find the numbers

A.M = 10

$\frac{a+b}{2} = 10$

a + b = 20

G.M = 8

$\sqrt{ab} = 8$

ab = 64

16,4

Q. Insert 3 numbers between 1 and 256 so that, the resulting sequence is a G.P

Let G_1, G_2, G_3 be the three numbers

1, G_1,G_2,G_3,256 is a G.P

a= 1

Fifth term is 256

$a\ r^4 = 256$

$r^4 = 256$

r= 4, -4

Take r = 4

$G_1 = 1x4 = 4$

$G_2 = 4x4 = 16$

$G_3 = 16 \text{ x } 4 = 64$

Chapter 10

Straight Lines

Slope of a line

Slope , m = tanθ where θ is the angle which the line makes with positive direction of X- axis in anti clockwise direction [$0^0 \leq \theta \leq 180^0$]

m = $\frac{y_2-y_1}{x_2-x_1}$

slope of X-axis = 0

slope of Y-axis = $not\ defined$

If the lines are parallel, $m_1 = m_2$

If the lines are perpendicular $m_1.m_2 = -1$

Q. Slope of a line is 5. Write the slope of another line parallel to the give line

5

Q. Slope of a line is $\frac{2}{3}$. Write the slope of another line perpendicular to the give line

$-\frac{3}{2}$

Q. Find the angle between X axis and the line joining (3,-1) and (4,-2)

m = $\frac{y_2-y_1}{x_2-x_1}$

= $\frac{-2+1}{4-3}$ = -1

Tanθ = -1

$\theta = 180^0 - 45^0 = 135^0$

Q. Find the value of x for which the points (x,-1),(2,1) and (4,5) are collinear

Slope of AB = $\frac{y_2-y_1}{x_2-x_1} = \frac{1+1}{2-x} = \frac{2}{2-x}$

Slope of BC = $\frac{5-1}{4-2}$ = 2

$\frac{2}{2-x}$ = 2

2 = 2(2-x)

1= 2-x

x=1

Different forms of equations of a straight line

Equation of x- axis : y = 0

Equation of y- axis : x = 0

Equation of a line parallel to x-axis : y =K

Equation of a line parallel to y-axis : x =K

Point slope form

The equation of a line passing through (x_1,y_1) and having slope m is

$$\mathbf{y - y_1 = m(x - x_1)}$$

This form is called point slope form

Two point form

The equation of a line passing through (x_1,y_1) and (x_2,y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Intercept form

The intercept form of a line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

a is the x intercept and b is the y intercept of the line

Q. find the equation of a line which makes equal intercepts on the axes and passing through (2,3)

Let x intercept = a

y intercept = a

$\frac{x}{a} + \frac{y}{a} = 1$

$\frac{x+y}{a} = 1$

x+y =a

The line passing through (2,3)

2+3 = a

a = 5

x+y = 5

Q. find the equation of a line passing through (2,2) and cutting off intercepts on the axes whose sum is 9

Let x intercept = a

Y intercept = 9 – a

$\frac{x}{a} + \frac{y}{9-a} = 1$

$\frac{x(9-a)+ya}{a(9-a)} = 1$

x(9-a) + ya = a(9-a)

Put x = 2, y = 2

2(9-a) +2a = a(9-a)

18-2a+2a = 9a – a^2

a^2-9a+18 = 0

(a-6)(a-3) = 0

a=6,3

Case 1

a=6

b=3

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{6} + \frac{y}{3} = 1$

Case 2

a=3

b=6

$\frac{x}{3} + \frac{y}{6} = 1$

Q. Find the equation of a line such that the midpoint of the line segment between the axes is (a,b)

X intercept = 2a

Y intercept = 2b

$\frac{x}{2a} + \frac{y}{2b} = 1$

$\frac{x}{a} + \frac{y}{b} = 2$

Slope intercept form

y = mx + c is the slope intercept form of a line where m is the slope and c is the y intercept

General equation of a line

The general equation of a line is Ax + By + C = 0

Slope = $\frac{-A}{B}$

x intercept = $\frac{-C}{A}$

y intercept = $\frac{-C}{B}$

Q. Find the slope, x intercept and y intercept of the line 2x+3y – 4 = 0

Slope = $\frac{-A}{B} = \frac{-2}{3}$

x intercept = $\frac{-C}{A}$ = $\frac{4}{2}$ = 2

y intercept = $\frac{-C}{B}$ = $\frac{4}{3}$

Q. Find the equation of a line perpendicular to x – 7y +5 = 0 and having x intercept 3

Slope of the given line = $\frac{-A}{B} = \frac{-1}{-7} = \frac{1}{7}$

Slope of the required line is -7 and passing through (3,0)

y– 0 = -7(x – 3)

y = -7x+21

7x+y-21 = 0

Angle between two lines

The acute angle θ between the lines is given by

$\tan\theta = \left|\frac{m_2 - m_1}{1+m_1m_2}\right|$ where m_1 and m_2 are slope of the lines

Q. Find the angle between the lines $\sqrt{3}$ x + y = 1 and x + $\sqrt{3}\ y = 1$

$m_1 = \frac{-A}{B} = -\sqrt{3}$

$m_2 = \frac{-1}{\sqrt{3}}$

$\tan\theta = \left|\frac{m_2 - m_1}{1+m_1m_2}\right|$

$= \left|\frac{\frac{-1}{\sqrt{3}}+\sqrt{3}}{1+\frac{-1}{\sqrt{3}}.-\sqrt{3}}\right|$

$= \left|\frac{\frac{-1+3}{\sqrt{3}}}{2}\right| = \left|\frac{\frac{2}{\sqrt{3}}}{2}\right| = \frac{1}{\sqrt{3}}$

$\theta = 30^0$

Q. Reduce the equation 4x + 3y – 12 = 0 into intercept form

4x+3y=12

$$\frac{4x}{12} + \frac{3y}{12} = 1$$

$$\frac{x}{3} + \frac{y}{4} = 1$$

X intercept = 3

Y intercept = 4

Chapter 11

Conic Sections

The conic sections (conics) are the curves obtained by intersecting a double - napped right circular hollow cone by a plane.

The four conics are

1. Circle
2. Ellipse
3. Parabola
4. Hyperbola

Circle

$(x-h)^2+(y-k)^2 = r^2$ is a circle with centre (h,k) and radius r

$x^2+y^2 = r^2$ is a cirle with centre (0,0) and radius r

Q. Find the equation of a circle with centre (2,2) and passes through (4,5)

Radius of the circle is the distance between (2,2) and (4,5)

r = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

=$\sqrt{(4-2)^2 + (5-2)^2}$ = $\sqrt{13}$

$(x-h)^2+(y-k)^2 = r^2$

$(x-2)^2+(y-2)^2 = \sqrt{13}^2$

$x^2+y^2-4x-4y-5 = 0$

The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Centre at (-g,-f)

r = $\sqrt{g^2 + f^2 - c}$

Q. Find the centre and radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

2g = 8

g = 4

2f = 10

f = 5

c = -8

Centre at (-g,-f)

Centre at (-4,-5)

r = $\sqrt{g^2 + f^2 - c}$

r = $\sqrt{4^2 + 5^2 + 8}$ = $\sqrt{49}$ = 7

Q. Find the centre and radius of the circle $2x^2+2y^2-x = 0$

$2x^2+2y^2-x = 0$

$x^2+y^2-\frac{x}{2} = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

2g = $\frac{-1}{2}$

g = $\frac{-1}{4}$

f = c = 0

Centre at (-g,-f)

Centre at ($\frac{1}{4}$,0)

r = $\sqrt{g^2 + f^2 - c}$

r = $\sqrt{(\frac{-1}{4})^2 + 0^2 - 0}$ = $\frac{1}{4}$

Q. Find the equation of a circle with radius 5 whose centre lies on x-axis and passes through

(2,3)

Let (h,0) be the centre

The distance between (h,0) and (2,3) is 5

$\sqrt{(h-2)^2 + (0-3)^2}$ = 5

h^2-4h+4+9 = 25

h^2-4h-12 = 0

(h-6)(h+2) =0

h=6,-2

Centre at (6,0) and radius 5

$(x-6)^2+(y-0)^2 = 5^2$

Centre at (-2,0) and radius 5

$(x+2)^2+(y-0)^2 = 5^2$

Q. Find the equation of the circle through the points (4,1), (6,5) and whose centre is on the line 4x+y = 16

Midpoint of the chord = = ($\frac{4+6}{2}$, $\frac{1+5}{2}$)= (5,3)

Slope of the chord = = $\frac{y_2-y_1}{x_2-x_1}$ = $\frac{5-1}{6-4}$ = 2

Slope of the perpendicular bisector of the chord is $\frac{-1}{2}$ and passing through (5,3)

$y-3 = \frac{-1}{2}(x-5)$

2(y-3) = -1(x-5)

2y-6 = -x+5

x+2y-11 = 0

4x+y-16 = 0

4x+8y-44 = 0

7y-28 = 0

7y = 28

y=4

x+2.4-11=0

x-3 = 0

x = 3

the centre of the circle is (3,4)

Radius is the distance between (3,4) and (4,1)

$r = \sqrt{(4-3)^2 + (1-4)^2}$

$= \sqrt{10}$

$(x-h)^2+(y-k)^2 = r^2$

$(x-3)^2+(y-4)^2 = \sqrt{10}^2$

Parabola

Parabola is the set of points in a plane that are equidistant from a fixed point and a fixed straight line.

The fixed point is called focus and the fixed line is called directrix

	vertex	focus	Directrix	Latus rectum	axis
$y^2 = 4ax$	(0,0)	(a,0)	x = -a	4a	X
$y^2 = -4ax$	(0,0)	(-a,0)	x = a	4a	X
$x^2 = 4ay$	(0,0)	(0,a)	y = -a	4a	Y
$x^2 = -4ay$	(0,0)	(0,-a)	y = a	4a	Y

Q. $y^2 = 12x$

$y^2 = 4ax$

4a = 12

a = 3

Vertex at (0,0)

Focus at (3,0)

Equation of directrix : x= -3

Length of the latus rectum = 4a= 12

Axis : x axis

Q. $x^2 = -16y$

$x^2 = -4ay$

4a = 14

a = 4

Vertex at (0,0)

Focus at (0,-4)

Equation of directrix : y = 4

Length of the latus rectum = 4a= 16

Axis : y axis

Q. Focus (0,-3), directrix y = 3

a = 3

$x^2 = -4ay$

$x^2 = -12y$

Q. Vertex (0,0), focus (-2,0)

a = 2

$y^2 = -4ax$

$y^2 = -8x$

Q. Vertex (0,0) passing through (2,3) and axis along x- axis

$y^2 = 4ax$

$3^2 = 4a.2$

9 = 8a

$a = \frac{9}{8}$

$y^2 = 4.\frac{9}{8}x$

$2y^2 = 9x$

Ellipse

An ellipse is the set of al points in a plane, the sum of whose distances from two fixed points in the plane is a constant

Two fixed points are called foci

$\mathbf{c^2 = a^2 - b^2}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Foci (±c,0)

Vertices (±a,0)

Length of the major axis = 2a

Length of the minor axis = 2b

Length of the latus rectum = $\frac{2b^2}{a}$

Eccentricity, e = $\frac{c}{a}$ (e < 1)

Distance between the foci = 2c

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Foci (0,±c)

Vertices (0,±a)

Q. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$a^2 = 16$

a = 4

$b^2 = 9$

b = 3

$c^2 = a^2 - b^2$

$= 4^2 - 3^2 = 7$

$c = \sqrt{7}$

Foci ($\pm\sqrt{7}$,0)

Vertices (±4,0)

Length of the major axis = 2.4 = 8

Length of the minor axis = 2.3 = 6

Length of the latus rectum = $\frac{2.3^2}{4} = \frac{9}{2}$

Eccentricity, e = $\frac{\sqrt{7}}{4}$

Q. $\frac{x^2}{4} + \frac{y^2}{25} = 1$

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$b^2 = 4$

b=2

$a^2=25$

a=5

$c^2 = a^2 - b^2 = 5^2 - 2^2 = 21$

c= $\sqrt{21}$

Foci (0,± $\sqrt{21}$)

Vertices (0,±5)

Length of the major axis = 2.5= 10

Length of the minor axis = 2.2 = 4

Length of the latus rectum = $\frac{2.2^2}{5} = \frac{8}{5}$

Eccentricity, e = $\frac{\sqrt{21}}{5}$ (e < 1)

Q. $4x^2+9y^2 = 36$

$\frac{4x^2}{36} + \frac{9y^2}{36} = 1$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a^2 = 9$

$a = 3$

$b^2 = 4$

$b = 2$

$c^2 = a^2 - b^2$

$= 3^2 - 2^2 = 5$

$c = \sqrt{5}$

Foci $(\pm\sqrt{5},0)$

Vertices $(\pm3,0)$

Length of the major axis = 2.3 = 6

Length of the minor axis = 2.2 = 4

Length of the latus rectum = $\frac{2b^2}{a} = \frac{2.2^2}{3} = \frac{8}{3}$

Eccentricity, e = $\frac{\sqrt{5}}{3}$

Find the equation of the ellipse satisfies the given conditions

Q. Vertices $(\pm5,0)$, foci$(\pm4,0)$

a=5, c=4

$c^2 = a^2 - b^2$

$4^2 = 5^2 - b^2$

$16 = 25 - b^2$

b=3

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$

Q. length of the minor axis 16, foci $(0,\pm6)$

2b = 16

B = 8

C = 6

$c^2 = a^2 - b^2$

$6^2 = a^2 - 8^2$

a = 10

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$\frac{x^2}{8^2} + \frac{y^2}{10^2} = 1$

Q. Ends of the major axis (±3,0), ends of the minor axis (0,±2)

a = 3, b = 2

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$

Q. major axis on x axis and passes through (4,3) and (6,2)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{4^2}{a^2} + \frac{3^2}{b^2} = 1$

$\frac{16}{a^2} + \frac{9}{b^2} = 1$

$\frac{6^2}{a^2} + \frac{2^2}{b^2} = 1$

$\frac{36}{a^2} + \frac{4}{b^2} = 1$

Put $\frac{1}{a^2}$ = u and $\frac{1}{b^2}$ = v

16u+9v=1

36u+4v=1

u = $\frac{1}{52}$ v = $\frac{1}{13}$

$\frac{1}{a^2} = \frac{1}{52}$

$a^2 = 52$

$\frac{1}{b^2} = \frac{1}{13}$

$b^2 = 13$

$\frac{x^2}{52} + \frac{y^2}{13} = 1$

Chapter 12

Introduction to Three Dimensional Geometry

Coordinates of origin : (0,0,0)

Coordinates of general point on x- axis : (x,0,0)

Coordinates of general point on y- axis : (0,y,0)

Coordinates of general point on z- axis : (0,0,z)

Coordinates of general point on XY-plane : (x,y,0)

Coordinates of general point on YZ-plane : (0,y,z)

Coordinates of general point on ZX-plane : (x,0,z)

x	y	z	octant
+	+	+	I
-	+	+	II
-	-	+	III
+	-	+	IV
+	+	-	V

- \+ - VI

\- - - VII

\+ - - VIII

Q. Write the octant in which (4,-2,3) lies

IV

Q. Write the octant in which (-1,-2,-3) lies

VII

Distance formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Q. Using distance formula show that the points P(-2,3,5), Q(1,2,3) and R(7,0,-1) are collinear

PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

= $\sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2}$

= $\sqrt{(3)^2 + (-1)^2 + (-2)^2}$ = $\sqrt{14}$

QR = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

= $\sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$

= $\sqrt{56}$ = $\sqrt{4 \times 14}$ = $2\sqrt{14}$

PR = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

= $\sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$

= $\sqrt{126}$

= $\sqrt{9 \times 14}$

= $3\sqrt{14}$

PQ + QR = PR

Points are collinear

Q. Find the equation of set of points P which are equidistant from A(1,2,3) and B(3,2,-1)

Let the coordinates of the point P be (x,y,z)

PA = $\sqrt{(x-1)^2 + (y-2)^2 + \ (z-3)^2}$

PB = $\sqrt{(x-3)^2 + (y-2)^2 + \ (z+1)^2}$

PA = PB

$PA^2 = PB^2$

$(x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$

$x^2-2x+1+y^2-4y+4+z^2-6z+9 = x^2-6x+9 + y^2-4y+4 + z^2+2z+1$

4x-8z = 0

x-2z=0

Chapter 13

Limits and derivatives

If f(x) is a polynomial function then

$$\lim_{x \to a} f(x) = f(a)$$

Q.

$$\lim_{x \to 3}(x+3) = 3+3$$

= 6

Q. $\lim_{x\to 2}\frac{4x+7}{2x+1} = \frac{4.2+7}{2.2+1}$

$= \frac{15}{5} = 3$

Q. $\lim_{x\to 1}\frac{x^2-1}{x-1} \quad \left(\frac{0}{0}\right)$

$= \lim_{x\to 1}\frac{(x+1)(x-1)}{(x-1)}$

$= \lim_{x\to 1}(x+1) = 1+1 = 2$

Q. $\lim_{x\to 2}\frac{x^3-4x^2+4x}{x^2-4}$

$= \lim_{x\to 2}\frac{x(x^2-4x+4)}{(x+2)(x-2)}$

$= \lim_{x\to 2}\frac{x(x-2)^2}{(x+2)(x-2)}$

$= \lim_{x\to 2}\frac{x(x-2)}{x+2} = \frac{2(2-2)}{(2+2)} = \frac{0}{4} = 0$

$$\lim_{x\to a}\frac{x^n - a^n}{x-a} = na^{n-1}$$

Q. $\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$

$= \lim_{x\to 1}\frac{x^{15} - 1^{15}}{x^{10} - 1^{10}}$

$= \frac{\lim_{x\to 1}\frac{x^{15}-1^{15}}{x-1}}{\lim_{x\to 1}\frac{x^{15}-1^{10}}{x-1}} = \frac{15.1^{14}}{10.1^{9}} = \frac{15}{10} = \frac{3}{2}$

$Q.$ $\lim_{x\to 0}\frac{(1+x)^5 - 1}{x}$

$\lim_{(1+x)\to 1}\frac{(1+x)^5 - 1^5}{(1+x)-1} = 5.1^{5-1} = 5.1 = 5$

Q. $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-1}{x}$

$= \lim\limits_{(1+x)\to 1}\frac{(1+x)^{\frac{1}{2}}-1^{\frac{1}{2}}}{(1+x)-1} = \frac{1}{2}\,.\,1^{\frac{1}{2}-1} = \frac{1}{2}$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\lim_{x\to 0}\frac{\tan x}{x} = 1$$

Q. $\lim\limits_{x\to 0}\frac{\sin 3x}{x}$

$\lim\limits_{3x\to 0}\frac{\sin 3x}{3x}\times 3 = 1\times 3 = 3$

$\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$

Q. $\lim\limits_{x\to 0}\frac{\sin 2x}{\tan 3x}$

$= \frac{\lim\limits_{2x\to 0}\frac{\sin 2x}{2x}\times 2x}{\lim\limits_{3x\to 0}\frac{\tan 3x}{3x}\times 3x} = \frac{2}{3}$

Q. $\lim\limits_{x\to 0}\frac{\sin 2x}{3x}$

$= \lim\limits_{2x\to 0}\frac{\sin 2x}{2x}\times\frac{2x}{3x} = \frac{2}{3}$

Q. $\lim\limits_{x\to 0}\frac{2x+\sin 3x}{\tan 7x+3x}$

$= \lim\limits_{x\to 0}\frac{2+\frac{\sin 3x}{x}}{\frac{\tan 7x}{x}+3}$

$= \frac{\lim\limits_{3x\to 0} 2+\frac{\sin 3x}{3x}\times 3}{\lim\limits_{7x\to 0}\frac{\tan 7x}{7x}\times 7+3} = \frac{5}{10} = \frac{1}{2}$

$$\lim_{x\to 0}\frac{e^x-1}{x} = 1$$

Q. $\lim_{x\to 0}\frac{e^{3x}-1}{x}$

= $\lim_{3x\to 0}\frac{e^{3x}-1}{3x}\times 3$

= 3

Q. $\lim_{x\to 0}\frac{e^{sinx}-1}{x}$

= $\lim_{x\to 0}\frac{e^{sinx}-1}{sinx}\times\frac{sinx}{x}$ = 1

$\lim_{x\to 0}\frac{\log(1+x)}{x}$ = 1

Q. $\lim_{x\to 0}\frac{\log(1+2x)}{x}$

= $\lim_{2x\to 0}\frac{\log(1+2x)}{2x}\times 2$ = 2

Q. $\lim_{x\to 0}\frac{\log(1+x^3)}{sin^3x}$

= $\lim_{x\to 0}\frac{\log(1+x^3)}{x^3}\times\frac{x^3}{sin^3x}$

= $\lim_{x^3\to 0}\frac{\log(1+x^3)}{x^3}\times\lim_{x\to 0}(\frac{x}{sinx})^3$ = 1

Differentiation

$\frac{dy}{dx}$ is the rate of change of y with respect to x

$\frac{d}{dx}(x^n)$ = nx^{n-1}

$\frac{d}{dx}$(x) = 1

$\frac{d}{dx}(\sqrt{x})$ = $\frac{1}{2\sqrt{x}}$

$\frac{d}{dx}(e^x)$ = e^x

$\frac{d}{dx}$(constant) = 0

$\frac{d}{dx}$(sinx)= cosx

$\frac{d}{dx}$(cosx) = -sinx

$\frac{d}{dx}$(tanx) = sec^2 x

$\frac{d}{dx}$(cotx) = - $cosec^2 x$

$\frac{d}{dx}$(secx) = secx tanx

$\frac{d}{dx}$(cosecx) = - cosecx cotx

$\frac{d}{dx}(x^2)$ = 2x

$\frac{d}{dx}(x^3)$ = 3 x^2

$\frac{d}{dx}(x^4) = 4x^3$

$\frac{d}{dx}$(f+g) = $\frac{d}{dx}$(f) + $\frac{d}{dx}$(g)

$\frac{d}{dx}$(f - g) = $\frac{d}{dx}$(f) - $\frac{d}{dx}$(g)

$\frac{d}{dx}$(kf) = k$\frac{d}{dx}$(f)

1. y = 2x+1

   $\frac{dy}{dx}$ = 2.1+0 = 2
2. y = $3x^2$+2x+5

   $\frac{dy}{dx}$ = 3.2x + 2.1 + 0

   = 6x+2
3. y = 3sinx + 4cosx – $5e^x$

$\frac{dy}{dx}$ = 3. Cosx + 4. -sinx – 5.e^x

= 3cosx-4sinx – 5e^x

Product rule

$\frac{d}{dx}$(fg) = f$\frac{d}{dx}$ (g)+ g $\frac{d}{dx}$(f)

1. y = xsinx

   $\frac{dy}{dx}$ = x$\frac{d}{dx}$(sinx) + sinx$\frac{d}{dx}$(x)

   = x.cosx + sinx.1

   = xcosx + sinx

2. y = x^2e^x

   $\frac{dy}{dx} = x^2 \frac{d}{dx}(e^x)$+ $e^x\frac{d}{dx}(x^2)$

   = $x^2.e^x + e^x.2x$

Quotient rule

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\times\frac{df}{dx} - f\times\frac{dg}{dx}}{g^2}$$

1. y = $\frac{x}{sinx}$

   $$\frac{dy}{dx} = \frac{sinx\times\frac{d(x)}{dx} - x\times\frac{d(sinx)}{dx}}{(sinx)^2} = \frac{sinx - x.cosx}{sin^2x}$$

Chapter 14

Mathematical Reasoning

A sentence is called mathematically acceptable statement if it is either true or false not both

Statements:

Two plus two equals four

8 is less than six

The sum of all interior angles of a triangle is 180^0

The product of -1 and 8 is 8

Not a statement:

Mathematics is difficult

The sides of a quadrilateral have equal length

Answer this question

Today is a windy day

He is a mathematics graduate

Kashmir is far from here

Open the door

While dealing with statements,we usually denote them by small letters p,q,r…

P: fire is always hot

q: All real numbers are complex numbers

Negation of a statement

The denial of a statement is called its negation

Q. Write the negation of the statement “Chennai is a city”

Chennai is not a city

or

It is false that Chennai is a city

The negation of p is denoted by $\sim p$

p. Every natural number is an integer

$\sim p$: It is false that every natural number is an integer

<u>contrapositive and converse</u>

The converse of $p \Rightarrow q$ is $q \Rightarrow p$

The contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$

Write the contrapositive and converse

Q. If a number is divisible by 9 then it is divisible by 3

<u>Converse</u>

If a number is divisible by 3, then it is divisible by 9

<u>Contrapositive</u>

If a number is not divisible by 3 then it is not divisible by 9

Q. If you are born in India, then you are a citizen of India

<u>Converse</u>

If you are a citizen of India then you are born in India

<u>Contrapositive</u>

If you are not a citizen of India, then you were not born in India

Chapter 16

Probability

Random experiments

In our day today life, there are so many experimental activities , the results may not be same when they are repeated under same conditions. Consider the following experiments

Tossing a coin

Throwing a die

We know the results of these experiments. But we are not sure which one of these results will come when it is executed. These kinds of experiments are called random experiments

The results of a random experiments are called outcomes

The set of all possible outcomes of a random experiment is called sample space and is denoted by s

Write the sample space of the following random experiments

1. Tossing a coin
   S = {H,T}
2. Tossing 2 coins
   S = {HH,HT,TH,TT}
3. Tossing 3 coins
   S = {HHH, HTT,THT,TTH,THH,HTH,HHT,TTT}

2^n is the number of elements in the sample space of tossing n coins

4. Throwing a die
   S= {1,2,3,4,5,6}
5. Throwing 2 dice
   S = {(1,1)…(6,6)}

6^n is the number of elements in the sample space of throwing n dice

6. A coin is tossed. If it shows a tail, we draw a ball from a box containing 2 red and 3 black balls. If it shows head, we throw a die
   Le R1,R2 be the red balls and B1,B2,B3 be the black balls
   S = {TR1,TR2,TB1,TB2,TB3,H1,H2,H3,H4,H5,H6}
7. A coin is tossed until a head comes up
   S = {H,TH,TTH,TTTH,…}

   Event

   The subset of a sample space is called event

   S is called sure event and ϕ is called impossible event

   Mutually exclusive events

   If A∩B = ϕ, then A and B are called mutually exclusive events

   Exhaustive events

   Two or more events are exhaustive if their union is the sample space

Probability of an event

Probability of the event a is denoted by P(A) and is defined as

P(A) = $\frac{n(A)}{n(S)}$

P(ϕ) = 0, P(S) = 1

0 ≤ P(A) ≤1

Q. A die is thrown. Find the probability of the following events

(i) getting an even number

S = {1,2,3,4,5,6}

A = {2,4,6}

P(A) = $\frac{n(A)}{n(S)} = \frac{3}{6}$

(II) getting an odd number

B = {1,3,5}

P(B) = $\frac{n(B)}{n(S)} = \frac{3}{6}$

(iii) Getting a prime number

C = {2,3,5}

P(C) = $\frac{n(C)}{n(S)} = \frac{3}{6}$

Q. 2 dice are thrown. Find the probability of the following events

(i) getting a sum of 8

S = {(1,1),….,(6,6)}

A = {(5,3),(3,5), (6,2),(2,6),(4,4)}

P(A) = $\frac{n(A)}{n(S)} = \frac{5}{36}$

(ii) getting a doublet

B = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}

P(B) = $\frac{n(B)}{n(S)} = \frac{6}{36}$

(iii)at least one of the dice shows 4

C= {(1,4),(4,1),(2,4),(4,2),(3,4),(4,3),(4,4),(5,4),(4,5),(6,4),(4,6)}

P(C) = $\frac{n(C)}{n(S)} = \frac{11}{36}$

Q. A bag contains 4 red and 5 black balls. A ball is drawn at random. Find the probability that

(i)the ball is red

$\frac{4_{C1}}{9_{C1}} = \frac{4}{9}$

(ii) The ball is black

$\frac{5_{C1}}{9_{C1}} = \frac{5}{9}$

Q. A bag contains 4 red and 5 black balls. 2 balls are drawn at random. Find the probability that

(i)the balls are red

$\frac{4_{C2}}{9_{C2}} = \frac{6}{36}$

(ii) The balls are black

$\frac{5_{C2}}{9_{C2}} = \frac{10}{36}$

(iii) The balls are of different colours

Select one ball from each colour

$\frac{4_{C1} \times 5_{C1}}{9_{C_2}} = \frac{20}{36}$

(iv)The balls are of same colour

P(2 balls are red) + P(2 balls are black)

$= \frac{6}{36} + \frac{10}{36} = \frac{16}{36}$

Q. A coin is tossed 3 times. Find the probability of the following events.

(i) exactly 2 tails

S = {HHH, HTT,THT,TTH,THH,HTH,HHT,TTT}
A = { HTT,THT,TTH }

P(A) = $\frac{3}{8}$

(ii) at least 2 tails

B = { HTT,THT,TTH,TTT }

P(B) = $\frac{4}{8}$

(iii) at most 2 tails

C = { HHH, HTT,THT,TTH,THH,HTH,HHT}

P(C) = $\frac{7}{8}$

Note:

Total number of cards = 52

26 red and 26 black

Diamond and heart are red , spade and club are black , each of which are of 13 cards

Number of face cards : 12

Q. 2 cards are drawn from a pack of 52 cards. Find the probability that

(i) the cards are red

$$\frac{26_{C2}}{52_{C2}}$$

(ii)the cards are of different colours

$$\frac{26_{C1} \times 26_{C1}}{52_{C_2}}$$

(iii) the cards are of same colour

P(2 cards are red) + P(2 cards are black)

$$\frac{26_{C2}}{52_{C2}} + \frac{26_{C2}}{52_{C2}}$$

(iv) The cards are of same suite

P(cards are diamond) + P(cards are heart) + P(cards are spade) + P(cards are club)

$$= \frac{13_{C2}}{52_{C2}} + \frac{13_{C2}}{52_{C2}} + \frac{13_{C2}}{52_{C2}} + \frac{13_{C2}}{52_{C2}}$$

$$= 4\times\frac{13_{C2}}{52_{C2}}$$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

If A and B are mutually exclusive, $P(A\cup B) = P(A) + P(B)$

$P(A) + P(A') = 1$

$P(A) = 1 – P(A')$

$P(A') = 1 – P(A)$

Q. $P(A) = \frac{4}{9}$ find P(notA)

$P(\text{notA}) = P(A') = 1-P(A) = 1-\frac{4}{9} = \frac{5}{9}$

Q. $P(A)= 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$ find

(i) $P(A\ or\ B) = P(A \cup B) = P(A) + P(B) - P(A\cap B) = 0.54 + 0.69 - 0.35 = 0.88$

(ii) $P(A'\cap B) = P(B) – P(A \cap B) = 0.69 – 0.35 = 0.34$

iii $P(A \cap B') = P(A) – P(A \cap B) = 0.54 – 0.35 = 0.19$

iv P(neither A nor B) $= P(A' \cap B') = P(A \cup B)' = 1-P(A \cup B) = 1 – 0.88 = 0.12$